### Time-Dependent Sensitivity Analysis of OECD Benchmark using BISON and RAVEN

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# **INTRODUCTION**

Developments in uncertainty quantification in nuclear simulations have decreased the computational cost required to perform accurate sensitivity analysis [1, 2, 3, 4]. Implementation of these methods in the RAVEN [5] framework allows additionally for time-dependent sensitivity analysis of uncertain input variables. By demonstration we consider an OECD benchmark case [6]. We propagate uncertainties in the input parameters using RAVEN operating on the BISON [7] fuels performance code. We then consider the time-evolution of the sensitivity of several output responses to the uncertain input parameters. We perform sensitivity analysis using time-based stochastic collocation for generalized polynomial chaos (SCgPC) and high-dimension model reduction (HDMR) [8].

## **METHODS**

The details of the OECD benchmark and its parameter uncertainties are described in [6]. The benchmark includes a fuel pin in a steady-state PWR with power transients over a 2000 day period. During this time there is a power ramp up, then 2 sharp drops in power after steady-state operation is reached. Uncertain parameters include fuel properties, boundary conditions, and geometries. We treat each of the uncertain inputs as independent parameters and consider here four responses: maximum centerline fuel temperature during the simulation, maximum creep strain experienced by the clad, fission gas release percent, and clad elongation. The independent uncertain input parameters and their distributions are given in Table I.

For propagation of uncertainty we make use of the high-dimension model reduction (HDMR) expansion [9],

$$u(Y) = u_0 + \sum_{n=1}^N u_n + \sum_{n_1=1}^N \sum_{n_2=1}^{n_1-1} u_{n_1,n_2} + \cdots, \qquad (1)$$

where u(Y) is the response as a function of inputs  $Y = (y_1, \ldots, y_N)$ , N is the dimensionality of the input space, and the components  $u_i$  are defined as

$$u_0 \equiv \int \cdots \int u(Y)\rho(Y)dY,$$
 (2)

$$u_1 \equiv \int \cdots \int u(Y)\rho(y_2, \cdots, y_N)dy_2 \cdots dy_N, \quad (3)$$

$$u_{1,2} \equiv \int \cdots \int u(Y)\rho(y_3,\cdots,y_N)dy_3\cdots dy_N, \quad (4)$$

and so forth, where  $\rho(Y)$  is the joint probability distribution function of *Y*. Each of the terms in Eq. 1 can be represented

using a generalized polynomial chaos expansion,

$$u(Y) \approx \sum_{k \in \Lambda} c_k \Phi_k(Y), \tag{5}$$

where  $\Phi_k$  are multidimensional orthonormal polynomials of order  $k = (k_1, \ldots, k_N)$  and  $\Lambda$  is a combination of multiindices corresponding to polynomial orders. Scalar coefficients  $c_k$  are approximated using sparse-grid collocation numerical integration [10],

$$c_k = \int \cdots \int u(Y) \Phi_k(Y) \rho(Y) dY \approx \sum_{\ell=1}^L w_\ell u(Y_\ell) \Phi_k(Y_\ell).$$
(6)

Sobol' sensitivity indices are obtained from the HDMR expansion as

$$S_i = \frac{\operatorname{var}[u_i]}{\operatorname{var}[u(Y)]}.$$
(7)

The accuracy of the Sobol' sensitivity indices is dependent on the order of polynomials used in the subset generalized polynomial chaos expansions. For this work, each subset is limited to first-order polynomials in each dimension, providing a linear understanding of the global sensitivities. While higher orders may reveal additional features, the linear expansion is much less expensive to calculate and provides a reasonable analysis of the uncertainty space.

Parameter	Mean	Std. Dev.
clad cond.	16	2.5
clad thick	6.7e-4	8.3e-6
clad rough	5e-7	1e-7
creep rate	1	0.15
fuel cond.	1	0.05
fuel dens.	10299.24	51.4962
fuel exp.	1e-5	7.5e-7
fuel radius	4.7e-3	3.335e-6
fuel swell	5.58e-5	5.77e-6
gap cond.	1	0.025
gap width	9e-5	8.33e-6
mass flux	3460	57.67
rod pressure	1.2e6	4e4
sys pressure	1.551e7	51648.3
power scaling	1	0.016667
Parameter	Low	High
inlet temp	558	564



Fig. 1: Response Mean Values

Time-dependent uncertainty analysis in RAVEN is performed using a snapshot approach: for each requested time step through the simulation, a HDMR expansion surrogate model is created for each response using the data reported from BISON. Interpolating between surrogates makes the collective time-dependent surrogate model. In this mode, it is critical to sample many time steps to provide accurate interpolation. Because the quadrature points needed to create the HDMR surrogates is the same at each time step, no additional BISON simulations are required to transition from steady-state to time-dependent uncertainty analysis.

## RESULTS

The evolution of the mean and variance of the four responses over burnup is given in Figs. 1-2. Each response is scaled linearly by the parameter shown in the legend. The nominal power shape in time is superimposed for reference.

In general variance increases as the transients are simulated; however, some drops in the variance of the max centerline temperature warrant attention. Immediately after each power drop, the max centerline temperature drops significantly as well. Because the variance is dominated by the system power near the transients, the system power scaling factor is the chief source of variance. Since the uncertain parameter is a scaling factor, a reduction in the total power results in a smaller variance, which is reflected in the reduction in variance for the max centerline temperature immediately after transients.

In Figs. 3-6, the evolution of sensitivities of various responses are shown with respect to increasing burnup. In addition, the power history used in the simulation is overlayed to provide insight in time-based changes. In each case, only the most significant uncertain inputs are shown for clarity. There are generally 4 significant events in the simulation cycle. The first occurs just after 1% fission per initial metal atom (FIMA), where the fuel has expanded enough to make contact with the clad. The remainder are near 0.02, 0.04, and 0.06 FIMA, where the system power drops.



Fig. 2: Response Variance Values



Fig. 3: Max Fuel Centerline Temperature

The max fuel centerline temperature is taken at the axial center of the fuel pin. Sensitivity to the fuel conductivity dominates over most of the variance history; however, system power has more impact near gradients in the power profile.

As expected, the clad creep rate is the most sensitive parameter for clad creep strain; however, it is interesting to note the rise and fall of the gap thickness as an important parameter in the middle of the burnup range.

Early in life the fission gas release is dependent on several parameters, which gives way to only the fuel conductivity and system power later.

The sensitivities in the variance of clad elongation have three distinct sections. At the beginning, clad elongation is perturbed most by clad conductivity, inlet temperature, and system power, with growing influence from fuel density. These are somewhat suddenly replaced by gap thickness, which then slowly trades places with clad creep rate over the remainder of the life cycle.



Fig. 4: Max Clad Creep Strain



Fig. 5: Fission Gas Release (Percent)



Fig. 6: Clad Elongation

### DISCUSSION

We have demonstrated how HDMR and SCgPC can be used in RAVEN to perform time-dependent uncertainty propagation analysis in codes modeling transient behavior. Reviewing the time-evolution of Sobol' sensitivities provides new methods in understanding the impact of uncertain input parameters as changes occur during the transient simulation. At small additional cost to static uncertainty propagation, transient analysis has valuable insights to offer.

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