# USNRC TECHNICAL TRAINING CENTER

**EXAMINATION COVER SHEET**

**Exam Title: Bayesian Inference in Risk Assessment (P-102) Date: April 12, 2019**

**Exam Points: 138**

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**INL NRC**

1. **This is an open text examination. The time limit for this examination is 2.5 hours. At the end of this time, the proctor will collect the examination materials even if the student is not finished. Passing score for this examination is 70%.**
2. **Answer all questions on the paper provided or saved Excel spreadsheets.**
3. **Read questions thoroughly. Ask the examination proctor for clarification if you do not understand the question or the meaning of any possible answers provided. State all assumptions. Show all work.**
4. **To obtain maximum credit for your answer, please write or print legibly.**
5. **Answer only the question asked. Credit may be deducted for any incorrect information included in the answer. If you are asked to list 3 items, only the first 3 items listed will be graded.**
6. **The value of each question is specified in the examination.**

**All work done on this examination is my own; I have neither given nor received aid.**

# Print Your Name Sign Your Name

**This examination was administered as an open text examination to the above student.**

# Start Time Stop Time Proctor's Initials Date

# Exam Points / 138 Grade: %

**Graded By:**

## Signature Date

**Bayesian Inference in Risk Assessment (P-102)**

### Open-Book Examination

**April 12, 2019**

**10 questions worth a total of 138 points**

1. (8 points) Provide an example of:

* (2 pts.) Two events (A, B) that are mutually exclusive

**A = diesel generator failure time in the [0,24] hours interval**

**B = diesel generator failure time greater than 24 hours**

* (2 pts.) Two events (A, B) that are independent

**A = diesel generator failure time in the [0,24] hours interval**

**B = feed-water motor operated valve failure on demand**

* (2 pts.) Two events (A, B) that are NOT mutually exclusive

**A = diesel generator failure time in the [0,24] hours interval**

**B = diesel generator failure time in the [12,36] hours interval**

* (2 pts.) Two events (A, B) that are NOT independent

**A = diesel generator failure time in the [0,24] hours interval**

**B = diesel generator fuel line failure time in the [0,24] hours interval**

1. (25 points) Provided two events (A and B) with their own probability values, Pr(A) = 0.28 and Pr(B) = 0.45:
   * (5 pts.) Determine Pr(A or B) assuming A and B are disjoint

**Pr(A or B) = Pr(A) + Pr(B) = 0.28 + 0.45 = 0.73**

* + (5 pts.) Determine Pr(A or B) assuming A and B are independent

**Pr(A or B) = Pr(A) + Pr(B) - Pr(A and B) = 0.28 + 0.45 - 0.28 \* 0.45 = 0.604**

* + (5 pts.) Determine Pr(A|B) assuming A and B are independent

**Pr(A|B) = Pr(A and B) / P(B) = Pr(A) Pr(B) / P(B) = Pr(A) = 0.28**

* + (5 pts.) Assuming Pr(A and B) = 0.15, determine Pr(A or B but not both)

**Pr(A or B but not both) = Pr(A or B) - Pr(A and B) = Pr(A) + Pr(B) – 2 \* Pr(A and B) =**

**= 0.478**

* + (5 pts) Assuming Pr(A and B) = 0.15, determine the probability of not being A AND not being B

**Pr(not being A AND not being B) = 1.0 - Pr(A or B) = 1.0 – (Pr(A) + Pr(B) - Pr(A and B)) =**

**= 0.396**

1. (20 points) The time occurrence of an event is described by an exponential distribution with parameter lambda = 0.25/hr.
   * (5 pts.) What are the mean value, median value, 5th, and 95th percentiles for the event occurrence time?

Mean = **1/lambda = 4.0 hr**

10th percentile = **0.21 hr (From DSW file)**

90th percentile = **11.98 hr (From DSW file)**

* + (5 pts.) What is the probability that the event will occur in the first 12 hours? Graphically show how this value has been determined.

**= 0.950 (From DSW file)**

* + (5 pts.) What is the probability that the event will occur after 12 hours? Graphically show how this value has been determined.

**= 1- 0.950 = 5. E-2**

* + (5 pts.) Build an Excel spreadsheet separate from the DSW to generate 100 Monte Carlo samples of event occurrence time. Use this spreadsheet to estimate the mean value and 90% credible interval for the event occurrence time. Save the spreadsheet on the shared folder as your name\_q3.xlsx . (Note that the answers will vary due to the low number of samples, the correct spreadsheet setup is graded).

Mean of event occurrence time = **3.95 (from a custom excel MC calculation)**

90% interval of event occurrence time = **[0.18, 12.04] (from a custom excel MC calculation)**

1. (10 points) Label the terms in Bayes’ Theorem given that is the parameter to estimate and is the set of collected data:
   * Pr(): **Prior**
   * Pr(): **Posterior**
   * Pr(): **Likelihood**
   * Pr(): **Probability of evidence**
   * Write the complete form of the Bayes Theorem using the notation listed above

**Pr() = Pr() \* Pr() / Pr()**

1. (9 points) For each of the events listed below (items a. b. and c.) provide:
   * The aleatory model (i.e., distribution) you would use for these variables
   * The specific probabilistic parameter
   * The type of data you would collect/need
2. (3 points) Failure to close of a motor operated valve on demand

**Aleatory model: Binomial**

**Parameter: p (probability of failure to close on demand)**

**Data = number of failures to close out of N demands**

1. (3 points) Failure to operate of an emergency gas turbine in the [0,24] interval

**Aleatory model: Exponential**

**Parameter: lambda (failure rate for event “Turbine fails to run”)**

**Data = operation times for gas turbine**

1. (3 points) Turbine related reactor trip events

**Aleatory model: Poisson**

**Parameter: lambda (rate of occurrence of turbine trip events)**

**Data = number of turbine related reactor trip events**

1. (10 points) Your test facility has collected data on fire suppression times in the turbine building. The following times (in minutes) have been recorded: 37, 56, 74, 24, 38, 22.
2. (5 points) Assume that these times are a random sample from an exponential distribution with parameter λ per minute. Using the Jeffreys noninformative prior for λ, calculate the posterior (i.e., updated) mean value of λ, the 5th and the 95th percentiles

Mean = **2.4 E-2 (from DSW file)**

5th percentile = **1.0 E-2 (from DSW file)**

95th percentile = **4.2 E-2 (from DSW file)**

1. (5 points) Rather than using the Jeffreys prior, assume the prior distribution for λ is given to you as a gamma(15, 125 minutes) distribution. Use this prior to calculate the posterior (i.e., updated) mean value of λ, the 5th and the 95th percentiles

Mean = **5.6 E-2 (from DSW file)**

5th percentile = **3.7 E-2 (from DSW file)**

95th percentile = **7.7 E-2 (from DSW file)**

1. (20 points) A power plant is currently buying controllers for motor operated valves from three different companies (A, B and C). The probability that a controller has a quality-related issue is 0.02, 0.07, 0.05 for company A, B and C respectively.
   1. (5 points) The plant is buying 150, 120 and 100 controllers from company A, B and C respectively. Given these 370 controllers, what is the expected number of controllers that has a quality-related issue?

**# expected defected controllers from A = 150 \* 0.02 🡪 about 3**

**# expected defected controllers from B = 120 \* 0.07🡪 about 8**

**# expected defected controllers from C = 100 \* 0.05 🡪 about 5**

**# expected defected controllers from A, B and C 🡪 about 16**

* 1. (5 points) Provided the 370 controllers bought on step a., if I randomly pick a controller, what is the probability that such controller has a quality-related issue?

**= 16/370 = 4.3 E-2**

* 1. (5 points) Provided the 370 controllers bought on step a., if I randomly pick a controller, what is the probability Pr(A) that such controller is provided from company A? Determine also Pr(B) and Pr(C).

**Pr(A) = 150/370 = 0.405**

**Pr(B) = 120/370 = 0.324**

**Pr(C) = 100/370 = 0.270**

* 1. (5 points) A controller is randomly chosen and tested before installation; the controller is found to have a quality-related issue. What is the probability that such controller is coming from company B?

**P(B|defective) = P(defective|B)\*P(B)/ P(defective) = 0.512**

**where:**

**P(defective|B) = 0.07**

**P(B) = 0.324**

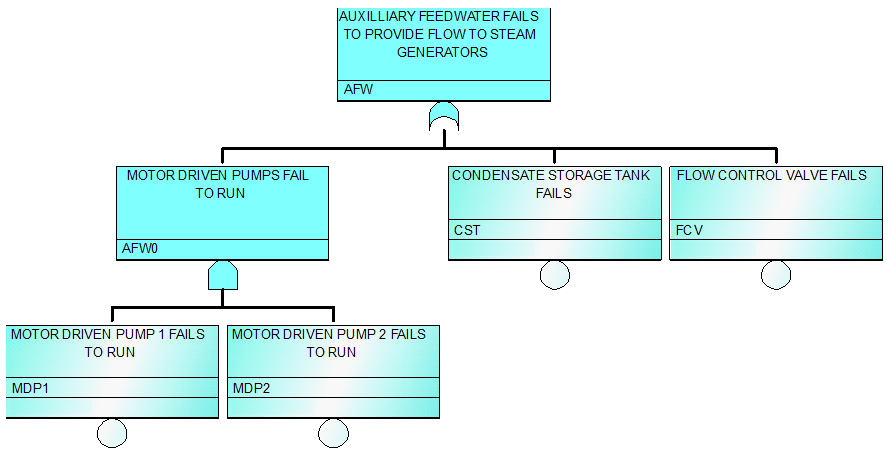
**P(defective) = P(defective|A)\*P(A) + P(defective|B)\*P(B) + P(defective|C)\*P(C) =**

**= 0.02 \* 0.405 + 0.07 \* 0.324 + 0.05 \*0.270 =**

**= 8.1 E-3 + 2.27 E-2 + 1.35 E-2 =**

**= 4.43 E-2**

1. (23 points) Consider the simplified Fault-Tree for the Auxiliary Feedwater (AFW) system of a PWR power plant:



To achieve success, the AFW system is required to run for a 24 hours mission. Data for the components is as follows:

* Condensate Storage Tank failure rate is 3.0 E-7/hour
* Each Motor Driven Pump failure rate is 2.5 E-4/hour
* Flow Control Valve failure rate is 7.4 E-5/hour.
  1. (1 pt.) What are the basic events of the provided Fault-Tree?

**MDP1 (Motor Driven Pump 1 fails to run), MDP2 (Motor Driven Pump 2 fails to run), CST (Condensate Storage Tank fails), FCV (Flow Control Valve fails)**

* 1. (3 pts.) Which failure model would you use for each component of the AFW system?

**Exponential**

* 1. (5 pts.) Determine the mission probability for each of the four basic events of the Fault-Tree.

**Pr(MDP1) = 1.0 – Exp(–lambda\_MDP1\*24) = 5.98 E-3**

**Pr(MDP1) = Pr(MDP2) = 5.98 E-3**

**Pr(CST) = 1.0 – Exp(–lambda\_CST\*24) = 7.2 E-6**

**Pr(FCV) = 1.0 – Exp(–lambda\_FCV\*24) = 1.77 E-3**

* 1. (8 pts.) What are the minimal cut sets (i.e., the combination of ways that the system can fail) of the provided Fault-Tree?

**Minimal cut sets = {MDP1 and MDP2, CST, FCV}**

* 1. (3 pts.) Using the rare event approximation (i.e., if each minimal cut set Ci has probability Pr(Ci), the rare event approximation for the system failure probability is Pr(system) ≈ Σ Pr(Ci)), estimate the probability the AFW system fails to run for the 24 hour mission time.

**Pr(system) = (5.98 E-3)^2 + 7.2 E-6 + 1.77 E-3 = 1.813 E-3**

* 1. (3 pts.) Is the rare event approximation really justified? Why?

**The small probability values associated to the set of minimal cut sets allow such approximation**

1. (8 points) Three diesel generators provide emergency power to a nuclear power plant. All of them have a failure probability of 0.01 to start on demand (only this failure mode is considered). One diesel generator is required to provide emergency power (the other two diesel generators are redundant).
   * (6 points) What is the probability that emergency power will NOT be provided when requested?

**Several ways can be used to solve this problem. Since the three diesel generators have identical failure probability a Binomial model can be used. Such Binomial model is designed to determine the probability that 0 diesel generators (out of 3) will operate.**

**Pr(failure) = Pr(0 operating generators) = Bin(0 out of 3, p=0.99) = 1.0 E-6**

* + (2 points) What is the probability that emergency power will be provided when requested?

**Pr(success) = 1.0 - Pr(failure) = 1.0 - 1.0 E-6**

1. (5 points) A turbine driven auxiliary feedwater pump has a lognormal distribution mean failure rate of 3.2E-3/hour and an error factor (EF) of 2.7. Update this distribution with collected data of 3 failures in 300 run hours. What is the updated mean, 5th percentile, and 95th percentile?

\*Note: The RADS calculator can be found at http://nrcoe.inl.gov/radscalc/

Mean = **5.11 E-3**

5th percentile = **2.0 E-3**

95th percentile **= 9.92 E-3**